Throughput and Fairness in Random Access Networks

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***Abstract*— This paper present an throughput analysis of log-utility and max-min fairness. Assuming all nodes interfere with each other, completely or partially, log-utility fairness significantly enhances the total throughput compared to max-min fairness since the nodes should have the same throughput in max-min fairness. The improvement is enlarged especially when the effect of cumulated interference from multiple senders cannot be ignored.**

I. Log-Utility and Max-Min Fairness

In this paper, we focus on dense wireless networks using random access protocols such as 802.11 LANs in offices and urban residential areas. We mainly consider slotted-Aloha systems but our analysis is simply extended for CSMA/CS networks with a single carrier-sensing range. In typical urban residential networks, one or few terminals are located closely to their access point and tend to capture a strong signal. The dense distribution of the access points, however, enlarges the effect of cumulative interference on frame reception.

For fair bandwidth allocation, there exist two famous fairness schemes: log-utility [1] and max-min fairness. Log-utility or proportional fairness has been well known as a flexible and useful abstraction for multiplexing scarce resources among users and applications. Max-min fairness, however, achieve the complete fair allocation, where all nodes have the same throughput.

Consider all nodes are within the single interference range. That is, any overlapped transmissions from the nodes completely collides and that results in transmission errors. Let N be the number of nodes. In this case, the nodes have the attempt probability $1/N$ in log-utility fairness [2] and the total throughput T_{LU} is given by:

$$\begin{aligned} T_{LU} &= \sum_i \frac{1}{N}(1 - \frac{1}{N})^{N-1}) \\ &= (1 - \frac{1}{N})^{N-1}. \end{aligned} \tag{1}$$

Note that T_{LU} is actually the same as that of max-min fairness. Let T_{MM} be the total throuhgput of max-min fairness. Since every node is in the same interference range, the nodes should have the same attempt probability. Let f be the attempt probability. the total throughput in a slotted-Aloha system is given by $Nf(1-f)^{N-1}$ and that is maximized at $f = 1/N$. Thus, if all nodes are in the same interference range, log-utility and max-min fairness achieves the same total throughput.

However, considering cumulative interference, max-min fair scheduling achieves lower aggregate throughput. In max-min fairness, nodes *interfering* must have the same throughput as the *interfered* neighbors [3]. Even when cumulative interference occurs infrequently, throughput of nodes outside of the interference range must drop to the same of nodes within the range. Figure 1 shows an example.

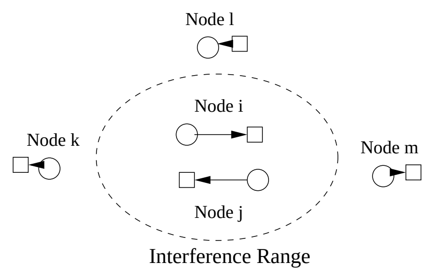

Fig. 1. 5 Node Pairs

In Figure 1, simultaneous transmissions from node k, l and m only interfere with node i and j. The log-utility fair allocation in Figure 1 achieves throughput at least twice that of max-min fairness. Consider the attempt probability of nodes. It is easy to show that node i and j have the same attempt probability. Let p be the attempt probability for node i and j. Node k, l and m also have the same. Let q be the probability.

As shown in our paper submitted Infocom 2007 [4], the attempt probability of node i for log-utility fairness is given by:

$$\frac{1 - f_i}{f_i} = \sum_h (1 - \frac{q_{h|i}}{q_h}), \tag{2}$$

where q_h is the success probability of transmissions from neighbor node h and $q_{h|i}$ is the conditional probability of successful transmissions from node h, given node i is transmitting. From Equation 2, p and q is obtained by $1/2$ and $1/\sqrt[3]{3}$ and the total throughput is around 2.41.

For max-min fairness, node throughput must be the same. The throughput of node i and j is given by $p(1-p)(1-q^3)$. For node k, l and m, throughput is q since they are not interfered at all. Note that regardless of q, the throughput is maximized at $p = 1/2$. From equation $q = (1-q^3)/$, we obtain $q = 0.246$ and the total throughput of max-min fairness is around

1.23. Thus, log-utility fairness achieves almost twice as total throughput in this example.

The more the number of outer nodes, the more the aggregate throughput obtained. While the max-min fair allocation only allow the same throughput as that in the inner interference range, log-utility fairness boosts the throughput of outer nodes by a lot, reducing the throughput of inner nodes a little.

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